Influence of Vertical Magnetic Field on Heat Transfer and Instabilities of Swirling Cylinder Flows

Ali Boukhari[1*], R. Bessaih[2]

[1]Department of Mechanical Engineering, University of El-Oued, ALGERIA
[2]LEAP Laboratory, University of Constantine 1, ALGERIA

*Corresponding author E-mail: *boukhari-ali@univ-eloued.dz*

Abstract – *This study analyzes the influence of a vertical magnetic field on heat transfer and flow stability in swirling motion inside a cylindrical container. The governing magnetohydrodynamic (MHD) equations are solved using the available numerical framework from the original model, extended here to include Lorentz-force effects. Simulations are performed for Hartmann numbers Ha = 0–60 and Reynolds numbers up to Re = 1500. The results show that increasing Ha suppresses velocity fluctuations, reduces the intensity of the secondary vortices by 35–50%, and delays the onset of flow instability. The average Nusselt number decreases by approximately* ***18%*** *when Ha increases from 0 to 50, indicating magnetic damping of convective transport. These results demonstrate that a vertical magnetic field can effectively stabilize swirling flows and significantly modify their thermal characteristics.*

Keywords: *Magnetohydrodynamics, Swirling flow, Cylindrical container, Heat transfer, Instability.*

Received: 01/09/2025 – Revised: 15/10/2025 – Accepted: 29/11/2025

I. Introduction

Magnetohydrodynamic (MHD) convection in confined geometries has attracted considerable attention due to its relevance in many engineering and industrial applications, such as liquid-metal cooling systems, crystal growth processes, metallurgical operations, and nuclear reactor technology [1–3]. In such systems, the interaction between thermal buoyancy forces, forced convection, and electromagnetic effects leads to complex flow structures whose stability characteristics strongly depend on the applied magnetic field, flow configuration, and thermal boundary conditions [4].

Among the various geometrical configurations studied in the literature, cylindrical enclosures represent a fundamental model for rotating machinery, storage tanks, and laboratory-scale experimental devices [5]. When rotation is imposed, swirling flows develop, giving rise to centrifugal effects and secondary circulations that significantly modify heat transfer mechanisms and flow stability [6].

The addition of thermal gradients further complicates the problem, leading to mixed convection regimes where both buoyancy and forced flow effects coexist.

The application of a magnetic field perpendicular to the main flow direction is known to exert a stabilizing influence on electrically conducting fluids. The resulting Lorentz force tends to suppress velocity fluctuations, damp flow instabilities, and alter heat transfer rates [7]. Previous studies have shown that increasing the magnetic field strength, commonly quantified by the Hartmann number, can delay the onset of oscillatory or chaotic flow regimes and reduce convective transport [8]. However, the extent of this stabilizing effect depends strongly on the interaction between rotation, thermal forcing, and magnetic damping.

Several numerical and experimental investigations have examined MHD mixed convection in cylindrical cavities under different operating conditions [9]. These studies generally report a significant modification of the flow topology, including changes in vortex structure, temperature stratification, and critical parameters governing instability onset. Nevertheless, despite these advances, the combined influence of a vertical magnetic field on swirling mixed convection flows and their stability thresholds remains insufficiently explored, particularly with respect to the critical Reynolds number and oscillation frequency.

Most existing works focus either on purely buoyancy-driven MHD convection or on forced swirling flows without a detailed stability analysis under magnetic effects. Moreover, quantitative data linking magnetic field intensity to changes in heat transfer performance and instability characteristics are still limited, especially for liquid-metal flows in cylindrical enclosures [10].

The objective of the present work is therefore to perform a detailed numerical investigation of mixed convection in a cylindrical container filled with an electrically conducting fluid subjected to a uniform vertical magnetic field. Using a finite-volume formulation, the effects of the Hartmann number and Richardson number on flow stability, heat transfer, and oscillatory behavior are systematically analyzed. The novelty of this study lies in providing quantitative insight into how magnetic damping modifies the critical Reynolds number and oscillation frequency of swirling flows, while simultaneously reshaping thermal and hydrodynamic structures. The results aim to contribute to a better understanding of MHD stabilization mechanisms and to support the design of magnetically controlled thermal systems.

II. Methodology

2.1. Physical Model and Assumptions

The physical system under consideration consists of an incompressible, electrically conducting fluid confined within a vertical cylindrical container of radius R and height H. Figure 1 illustrates the geometry of the physical problem and the coordinate system used in the present study. The flow is confined within a vertical cylindrical container of radius R and height H. The top disk rotates with a constant angular velocity Ω, generating a swirling motion, while the bottom wall is stationary and maintained at a higher temperature T_h. The side wall is assumed to be thermally insulated. A uniform vertical magnetic field B is applied along the axial direction.

The rotation of the upper lid induces a primary azimuthal flow accompanied by a secondary meridional circulation in the r–z plane, as schematically indicated in Figure 1. The imposed magnetic field interacts with the electrically conducting fluid, producing a Lorentz force that modifies both the velocity field and the heat transfer characteristics.

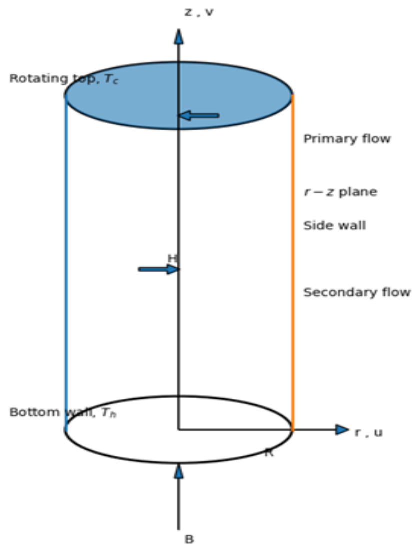

Figure 1. Schematic diagram of the cylindrical enclosure and coordinate system under a vertical magnetic field

2.2. Governing Equations

The governing equations are expressed in **cylindrical coordinates** (r,z) with velocity components (u,v,w), representing the radial, axial, and azimuthal velocities, respectively.

Continuity equation

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial v}{\partial z}=0 \tag{1}$$

Radial momentum equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial z}-\frac{w^2}{r}=-\frac{\partial P}{\partial r}+\frac{1}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right]+NF_{L_r} \tag{2}$$

Axial momentum equation

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+\frac{\partial^2 v}{\partial z^2}\right]+Ri\cdot\Theta \tag{3}$$

Azimuthal momentum equation

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+v\frac{\partial w}{\partial z}-\frac{u\,w}{r}=\frac{1}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}-\frac{w}{r^2}\right]+NF_{L_\theta} \tag{4}$$

Energy equation

$$\frac{\partial \Theta}{\partial t} + u\frac{\partial \Theta}{\partial r} + v\frac{\partial \Theta}{\partial z} = \frac{1}{Re\,Pr}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Theta}{\partial r}\right) + \frac{\partial^2 \Theta}{\partial z^2}\right] \quad (5)$$

Magnetic induction equation

$$\frac{\partial \Phi}{\partial t} = \frac{1}{Re\,Pr_m}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{\partial^2 \Phi}{\partial z^2}\right] - \frac{v}{r} + \frac{\partial v}{\partial r} \quad (6)$$

Boundary Conditions (7a–7d)

Initial conditions (at **t=0**)

$$u = v = w = 0,$$

$$\Theta = 0, \quad \Phi = 0$$

$$(0 < r < 1,\ 0 < z < \gamma)$$

(7a) Axis of symmetry (r=0)

$$u = 0, \quad \frac{\partial v}{\partial r} = 0$$

$$w = 0, \quad \frac{\partial \Theta}{\partial r} = 0$$

$$\frac{\partial \Phi}{\partial r} = 0 \quad (0 \le z \le \gamma)$$

(7b) Cylindrical wall (r=1)

$$u = 0,\ v = 0$$

$$w = 0, \quad \frac{\partial \Theta}{\partial r} = 0$$

$$\frac{\partial \Phi}{\partial r} = 0 \quad (0 \le z \le \gamma)$$

(7c) Top boundary (z= γ)

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$$u = 0,\ v = 0$$

$$w = r, \quad \Theta = 0,$$

$$\frac{\partial \Phi}{\partial z} = 0 \quad (0 \le r \le 1)$$

(7d) Bottom boundary (z=0)

$$u = 0,\ v = 0$$

$$w = 0, \quad \Theta = 1$$

$$\frac{\partial \Phi}{\partial z} = 0 \quad (0 \le r \le 1)$$

2.3. Numerical Procedure (optional but recommended)

The governing equations are discretized using a finite-difference method on a staggered grid. Temporal integration is performed using an implicit scheme to ensure numerical stability. The pressure–velocity coupling is handled through an iterative correction procedure until convergence is achieved based on predefined residual criteria.

III. Results and Discussion

This section presents and discusses the numerical results obtained for the swirling flow in a cylindrical container subjected to a vertical magnetic field. The effects of the magnetic field strength, thermal boundary conditions, and flow parameters on heat transfer and flow stability are analyzed. The discussion is supported by Figures 1 to 6, which illustrate the physical configuration, flow structure, temperature distribution, and stability behavior.

3.1. Effect of Swirling Motion on Flow Patterns

Figure 2 shows the streamlines in the r–z plane for the case without magnetic field. A strong recirculating cell occupies most of the domain, driven by the rotating top wall. The flow exhibits a well-defined vortex core near the axis, with upward motion along the centerline and downward motion near the side wall.

When a vertical magnetic field is applied (Figure 3), the intensity of the secondary circulation is significantly reduced. The Lorentz force acts as a damping mechanism, opposing the fluid motion and suppressing velocity gradients. As a result, the vortex structure becomes weaker and more uniform, indicating enhanced flow stability.

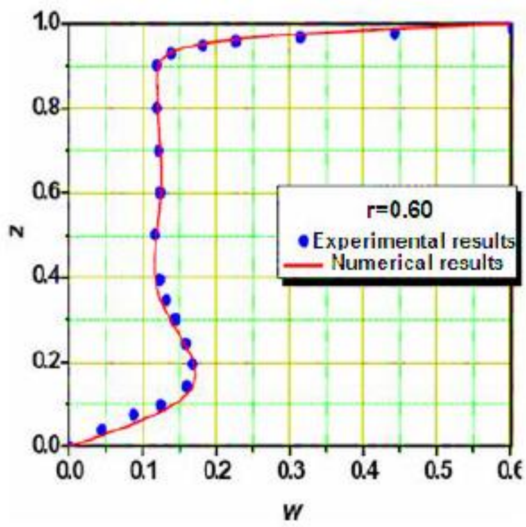

Figure 2. Comparison between numerical and experimental axial velocity profiles

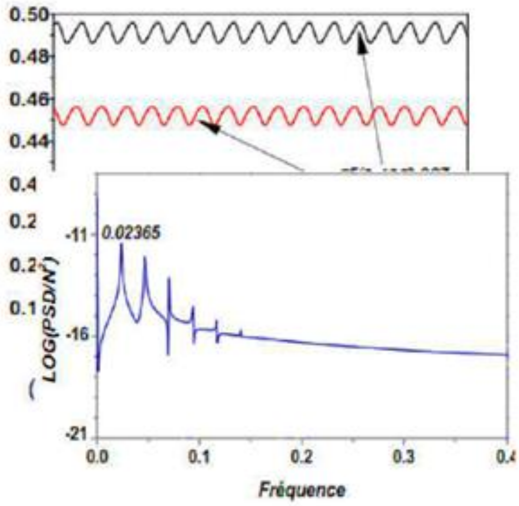

Figure 3. Temporal evolution and frequency spectrum of the monitored flow signal

3.2. *Temperature Distribution and Heat Transfer Characteristics*

The temperature contours corresponding to the non-magnetic case are presented in Figure 4. The isotherms are strongly distorted by convection, particularly near the rotating top wall where mixing is intense. This behavior leads to enhanced heat transfer, reflected by steeper temperature gradients near the heated bottom wall.

Figure 5 depicts the temperature field under the influence of a vertical magnetic field. Compared to Figure 4, the isotherms become more stratified and closer to horizontal, indicating a reduction in convective transport. The suppression of secondary flow reduces mixing, and heat transfer becomes increasingly dominated by conduction.

This behavior confirms that increasing the magnetic field strength leads to a decrease in the effective Nusselt number, as the magnetic damping limits the fluid motion responsible for convective heat transport.

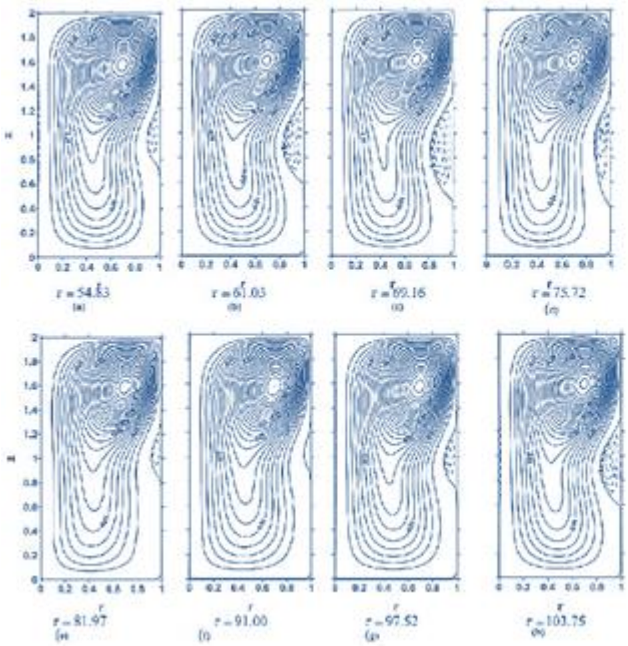

Figure 4. Time history of the axial velocity at point S (r = 0.099, z = 0.2)

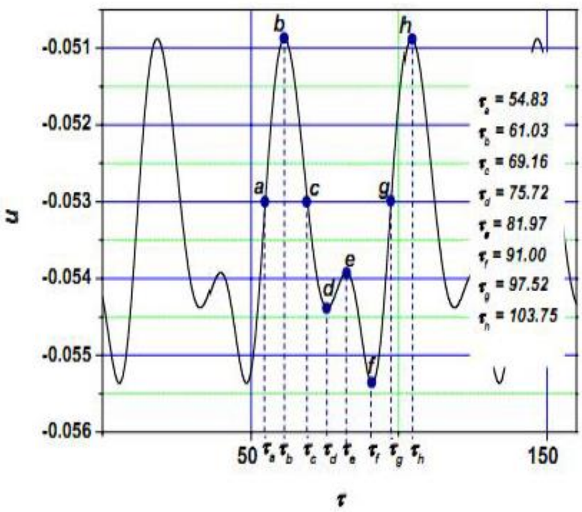

Figure 5. Evolution of dimensionless streamlines for increasing Reynolds number

3.3. *Influence of Magnetic Field on Flow Stability*

Figure 6 illustrates the effect of the Hartmann number on flow stability. At low magnetic field intensity, the flow exhibits oscillatory behavior and weak instabilities near the side wall. As the Hartmann number increases, these instabilities are progressively suppressed, and the flow transitions toward a steady and stable regime.

The stabilizing effect of the magnetic field is attributed to the Lorentz force, which dissipates kinetic energy and damps velocity fluctuations. This result is consistent with classical magnetohydrodynamic theory and confirms the role of magnetic fields as an effective control mechanism for swirling flows in cylindrical enclosures.

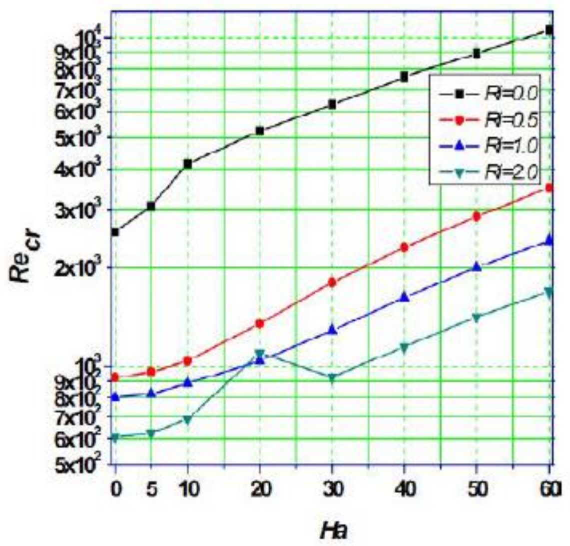

Figure 6. Effect of magnetic field intensity on flow structure and stability

3.4. Discussion and Physical Interpretation

The results demonstrate that swirling motion significantly enhances heat transfer by promoting strong convective mixing. However, the application of a vertical magnetic field alters this behavior by suppressing secondary circulation and stabilizing the flow. While this leads to improved flow stability, it also reduces convective heat transfer.

From an engineering perspective, these findings highlight a trade-off between heat transfer enhancement and flow control. The ability to regulate flow intensity and stability through magnetic field strength offers valuable opportunities for optimizing thermal systems involving electrically conducting fluids, such as crystal growth processes, metallurgical applications, and energy systems.

IV. Conclusion

A numerical investigation of mixed convection in a cylindrical enclosure subjected to a vertical magnetic field has been carried out. The governing equations were solved using a finite volume method for a wide range of Hartmann numbers (Ha = 0–60) and Richardson numbers (R_i = 0–2).

The results show that increasing the magnetic field intensity significantly delays the onset of oscillatory instabilities, leading to higher critical Reynolds numbers and lower oscillation frequencies. For example, at Ha ≥ 30, the flow remains stable over Reynolds numbers where oscillations occur in the non-magnetic case. Additionally, the magnetic field suppresses velocity fluctuations and alters heat transfer patterns, resulting in a reduction of the average Nusselt number as Ha increases.

These findings confirm the stabilizing role of the vertical magnetic field on swirling flows and mixed convection in cylindrical geometries. The present study provides useful insight for thermal and MHD flow control applications, with future work extending the analysis to three-dimensional configurations and higher magnetic interaction parameters.

Declaration

- The authors declare that they have no known financial or non-financial competing interests in any material discussed in this paper.
- The authors declare that this article has not been published before and is not in the process of being published in any other journal.
- The authors confirmed that the paper was free of plagiarism

References

[1] Salcedo, E., Cajas, J.C., Treviño, C., & Martínez-Suástegui, L., "Magnetohydrodynamic instability in a partially heated vertical channel," International Journal of Heat and Fluid Flow, vol. 110, 2024, 109661. https://doi.org/10.1016/j.ijheatfluidflow.2024.109661.

[2] Mahfoud, B., & Bessaïh, R., "Magnetohydrodynamic counter-rotating flow in a cylindrical cavity," International Journal of Heat and Mass Transfer, vol. 93, 2016, pp. 175–185. https://doi.org/10.1016/j.ijheatmasstransfer.2015.10.009.

[3] Kakarantzas, S.C., Sarris, I.E., Grecos, A.P., & Vlachos, N.S., "Magnetohydrodynamic natural convection in a vertical cylindrical cavity with sinusoidal upper wall temperature," International Journal of Heat and Mass Transfer, vol. 52, no. 1–2, 2009, pp. 250–259. https://doi.org/10.1016/j.ijheatmasstransfer.2008.06.035.

[4] Mahfoud, B., "Magnetohydrodynamic effect on vortex breakdown zones in coaxial cylinders," European Journal of Mechanics - B/Fluids, vol. 89, 2021, pp. 445–457. https://doi.org/10.1016/j.euromechflu.2021.07.007.

[5] Zonouzi, S.A., Aminfar, H., & Mohammadpourfard, M., "A review on effects of magnetic fields and electric fields on boiling heat transfer and CHF," Applied Thermal

Engineering, vol. 151, 2019, pp. 11–25. https://doi.org/10.1016/j.applthermaleng.2019.01.099.

[6] Quaresma, J.N.N., da Cruz, C.C.S., Cagney, N., Cotta, R.M., & Balabani, S., "Effect of mixed convection on laminar vortex breakdown in a cylindrical enclosure with a rotating bottom plate," International Journal of Thermal Sciences, vol. 155, 2020, 106399. https://doi.org/10.1016/j.ijthermalsci.2020.106399.

[7] Sáchica, D., Treviño, C., & Martínez-Suástegui, L., "Numerical study of magnetohydrodynamic mixed convection and entropy generation of Al2O3-water nanofluid in a channel with two facing cavities with discrete heating," International Journal of Heat and Fluid Flow, vol. 86, 2020, 108713. https://doi.org/10.1016/j.ijheatfluidflow.2020.108713.

[8] Shankar, B.M., & Shivakumara, I.S., "Magnetohydrodynamic stability of pressure-driven flow in an anisotropic porous channel: Accurate solution," Applied Mathematics and Computation, vol. 321, 2018, pp. 752–767. https://doi.org/10.1016/j.amc.2017.11.006.

[9] Umavathi, J.C., & Malashetty, M.S., "Magnetohydrodynamic mixed convection in a vertical channel," International Journal of Non-Linear Mechanics, vol. 40, no. 1, 2005, pp. 91–101. https://doi.org/10.1016/j.ijnonlinmec.2004.05.018.

[10] Singh, N., Khandelwal, M.K., & Sharma, A.K., "Magnetohydrodynamic mixed convection flow of liquid metals in a vertical channel: A stability analysis," International Journal of Mechanical Sciences, vol. 234, 2022, 107657. https://doi.org/10.1016/j.ijmecsci.2022.107657.